JOURNAL OF MATERIALS SCIENCE 9 (1974) 1521–1535

Review
Thermoelasticity, pseudoelasticity and the memory effects associated with martensitic transformations

Part 1 *Structural and microstructural changes associated with the transformations*

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The literature pertaining to the microstructural and crystallographic features of thermoelasticity, pseudoelasticity and memory effects associated with martensitic transformations is reviewed. The interrelations between the various effects are described. An introduction to the mechanical behaviour and the thermodynamic parameters is given.

1. Introduction

Martensitic transformations can be induced by the application of stress as well as by changes in temperature. This interchangeability of temperature and stress as variables affecting the transformation is due to two effects:

(1) the free enthalpy of the matrix and product phase and thus their equilibria depend not only on variations in temperature and composition but also on stress (the influence of shear stresses is dominant; volume stresses become significant only in the ultra-high pressure range e.g. [1]);

(2) the nucleation and growth processes are associated with shear strains and these will interact with stresses acting within, or applied to, the specimen.

Moreover, the thermodynamic (1) and the kinetic (2) effects are strongly dependent on the directions of stresses with respect to the lattice orientations. This nature of martensitic transformation leads to the effects which are treated on their common basis in this paper: thermoelasticity, pseudoelasticity, the shape memory effect and the two-way shape memory effect. Our main goal is to compile experimental and theoretical results thus far obtained in this field and to relate the various phenomena to the principal properties associated with martensitic transformations.

First we shall describe briefly the transformation and the deformation phenomena which will be treated.

A *thermoelastic martensitic transformation* is realized if martensite forms and grows continuously as the temperature is lowered, and shrinks and vanishes continuously as the temperature is raised as shown in Fig. 1. Fig. 2a gives a schematic representation of this transformation by indicating the increase in internal stresses, σ_i (and, thus, of stored elastic energy) and the fractional increase in the amount of martensite as measures of the progress of transformation, and the decrease of these properties due to the reverse transformation. There is no sudden appearance or disappearance of large groups of plates (burst). The trans-

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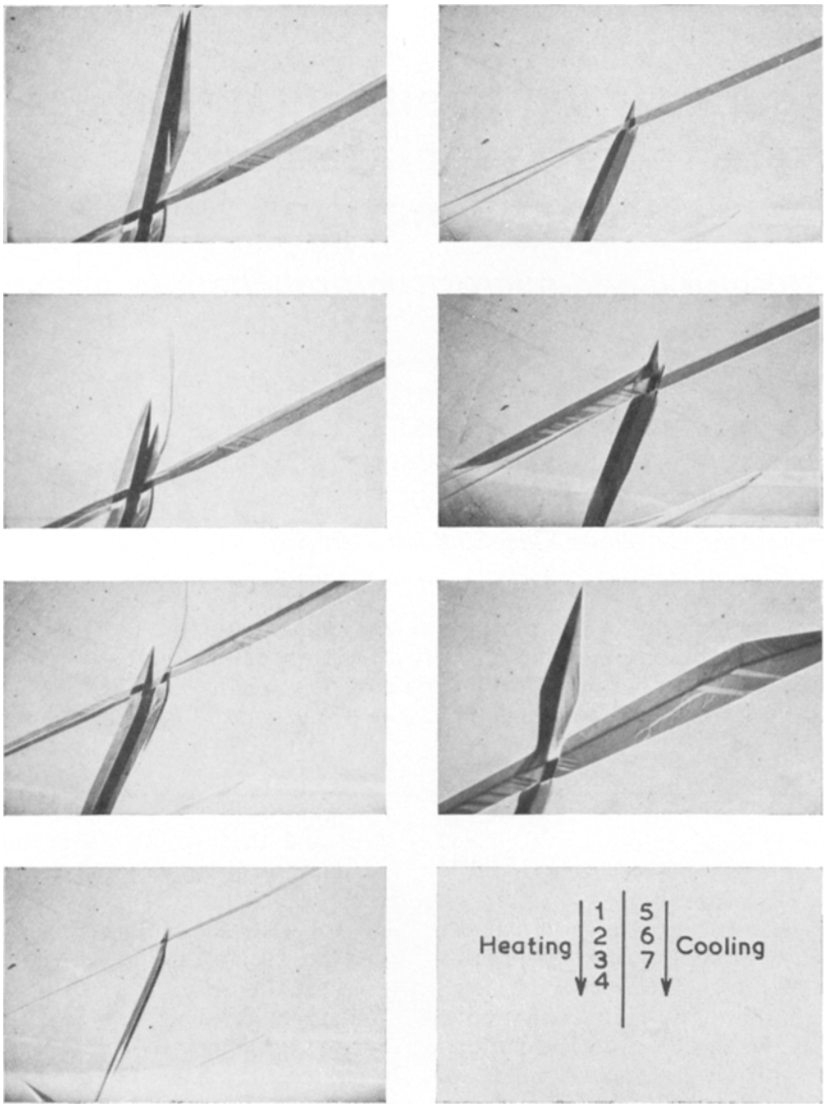

Figure 1 Thermoelastic behaviour in AgCd showing the growth of self-accommodating groups.

formation proceeds essentially in equilibrium between the *chemical driving energy* of the transformation and a *resistive energy* whose dominating component is the stored elastic energy. Thus the transformation is defined as being *elastic* or *degenerate elastic* if during the growth process the chemical driving energy is always in equilibrium with the resistive energy. The term *degenerate elastic transformation* refers to a process which shows small local discontinuities or jumps during the growth of a martensite plate. The transformation is said to be *spontaneous* or *burst* whenever the chemical driving energy exceeds the resistive energy to a great extent. Once the transformation starts, the growth or catalytic nucleation cannot be suppressed by external influences.

The *pseudoelastic behaviour* is a complete mechanical analogue to the thermoelastic transformation. In this case the transformation proceeds continuously with increasing applied stress, σ_a (and external strain ϵ) and is reversed continuously when the stress is decreased. Fig. 3 shows an example of this behaviour in terms of a

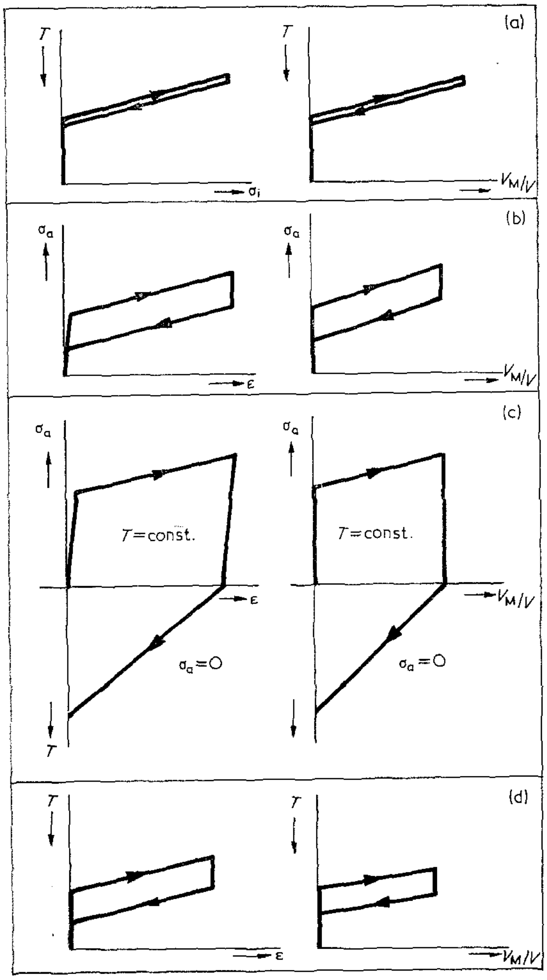

Figure 2 (a) Schematic representation of the formation and reverse transformation of thermoelastic martensite as a function of temperature; σ_i represents the internal stresses arising by the transformation, V_M/V the fractional volume of the martensite product. (b) Schematic representation of pseudoelasticity. Martensite forms as the applied stress is increased producing the macroscopic strain. Release of stress causes the martensite to revert to the parent phase. (c) Schematic representation of shape memory effect. Martensite is formed by straining but remains stable upon removal of the external stress; it is reverted to the parent phase on heating. (d) Schematic representation of two-way memory effect. On cooling macroscopic strain is produced without the application of an external stress; the strain disappears again on increasing the temperature.

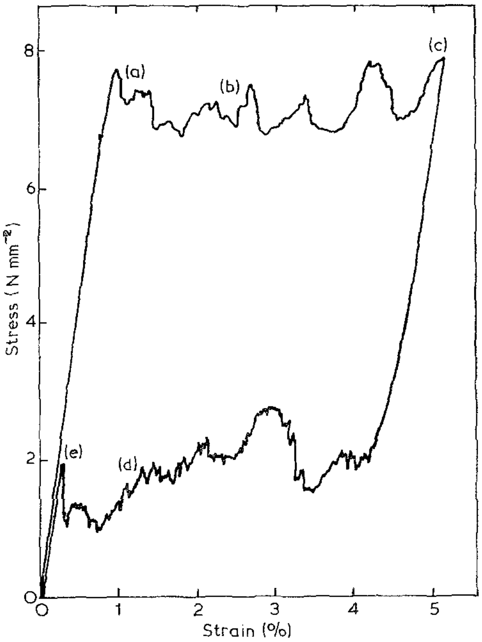

Figure 3 Tensile curve showing the pseudoelasticity in a Ag–Cd alloy. The upper part of the curve corresponds to the transformation, the lower part corresponds to the reverse transformation. The letters on the figure correspond with the microstructures given in Fig. 6 [30].

stress-strain curve. The "plastic" strain is caused by the shape strain accompanying the formation of martensite. Therefore, recovery occurs when the transformation is reversed. In the left-hand diagram of Fig. 2b the stress-strain curve of Fig. 3 is drawn schematically whereas the diagram on the right in Fig. 2b indicates the concomitant change in fractional martensite volume. It will be noted that in Fig. 2b a hysteresis is shown to occur. This behaviour is due to the interaction of several thermodynamic and kinetic factors. The treatment of these with respect to all phenomena of present interest is the subject of Part 3 of this series.

The *shape memory effect* arises if a macroscopic deformation is accompanied, as before, by a martensitic transformation which is not reversed by removing the applied stress; in a second step the reverse transformation and a concomitant reversal of the macroscopic deformation are induced by heating. An example is shown in Fig. 4. Fig. 2c shows this behaviour schematically; the left-hand diagram pertains to the performance of a shape memory experiment in a tensile test. The upper half represents the response of the specimen to the isothermal increase and decrease of the applied stress, the lower half to the effect of the subsequent temperature increase. The right-hand diagram

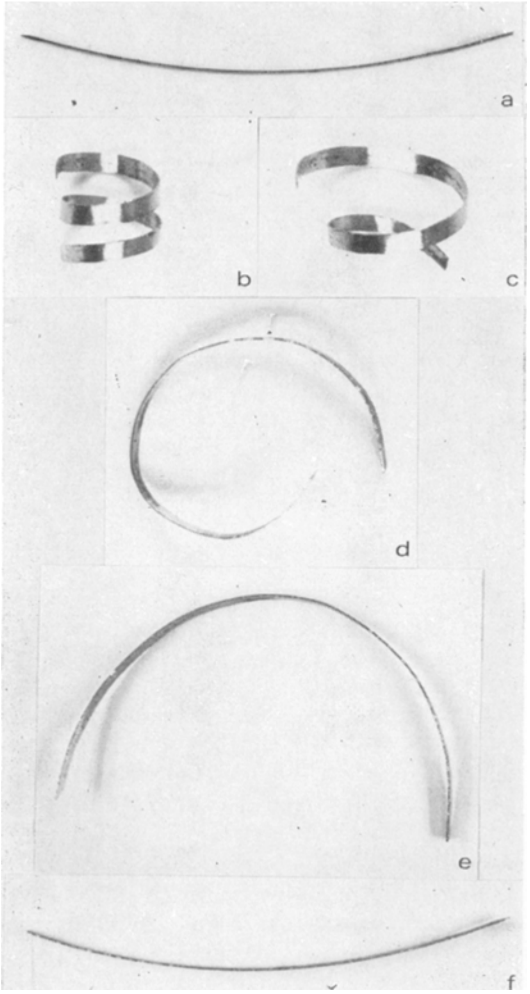

Figure 4 (a) A series of photographs showing the shape memory effect (a) the undeformed state, (b) after deformation and before the shape recovery, (c), (d) and (e) during the shape recovery, (f) after the shape recovery has been completed.

indicates the change in fractional amount of martensite again.

The idealized curves of Fig. 2a to c indicate that close interrelations of the processes exist. It will be recognized that if the hysteresis in the case of pseudoelastic transformation is so large that the reverse transformation is incomplete at $\sigma_a = 0$, the residual martensite can be reverted by heating, i.e. by employing the shape memory effect. This exhibits the interchangeability of stress and temperature as state variables. Thermodynamically the relation is given by the Clausius-Clapeyron equation in a form modified to account for shear strain as well as volume changes. Treatments of martensitic transformations on this basis are not new. They have been employed to compute the transformation enthalpy from the changes in transformation temperatures (both forward and reverse) with applied stress [2, 3].

Up to this point we have implied that pseudoelasticity and the shape memory effect were associated with a martensitic transformation. The same phenomena can occur even when the specimen is fully martensitic at the outset. In this case the reversible fraction of the resistive energy is stored by *reorientation* of the martensite plates and/or their internal structure (e.g. twins). Thus, pseudoelasticity and the shape memory effect may be associated with a martensitic transformation, a reorientation of a martensitic structure or a combination of both.

The final process to be described is the *two-way shape memory effect* (Fig. 2d). This term refers to a reversible shape change accompanying thermally induced martensite formation and reverse transformation. Normally, a thermal martensitic transformation proceeds by the formation of random numbers of martensite plates of each orientation variant in a sufficiently large sample such that no macroscopic shape change occurs apart from a volume change. In the case of a preferred orientation distribution of nuclei the transformation leads to a corresponding limitation of orientation variants which, in turn, causes an anisotropic macroscopic shape change. If the nuclei are not destroyed upon reverse transformation or by high-temperature annealing, the transformation-induced shape change can be repeated through successive thermal cycles. An extreme example of the two-way shape memory effect is the reversible shape change accompanying a single interface martensitic transformation due to the presence of only one nucleus. However, preferred orientations of persistent nuclei can, also, be induced by plastic deformation of the matrix, of the martensite or by inducing the first transformation under uniaxially applied stress.

The martensitic transformations and the reorientation of martensitic structures by applied stresses may be treated as deformation processes which are closely related to twinning. The treatment of continual mechanical twinning by Bolling and Richman [4] taking into account a free energy change is, in fact, identical to a treatment of the growth process of a diffusionless structural transformation under stress as Laves has pointed out [5]. Its application to martensitic transformations in Fe–Ni–C alloys [6] has shown its potential. In the present papers a treatment on this general basis, but with extended attention to structural details, leads to a more complete assessment of the interactions of stress with martensitic transformations.

The shape memory effect and the other processes discussed are not recent discoveries. The continuous appearance and disappearance of martensite with falling and rising temperature, i.e. the thermoelastic behaviour, has been observed by Greninger and Mooradian in a Cu–Zn alloy as early as 1938 [7], but the first detailed study of this effect was only published 11 years later by Kurdyumov and Khandros [8]. Pseudoelasticity and the shape memory effect were implicitly studied by observations of length changes by temperature variation and concomitant martensitic transformation under constant load by Scheil in Fe–Ni alloys [9] and by Hornbogen and Wassermann in Cu–Zn [10]. The shape memory effect was also observed in AuCd by Chang and Read [11]. Pseudoelasticity due to reorientation and transformation was recognized by Burkart and Read in In–Tl [2] and by Chang and Read in AuCd [11] and was studied extensively by Birnbaum and Read [12]. The recent revival of interest in the shape memory effect was instigated by the investigations on NiTi by Buehler and co-workers (e.g. [13]) who also observed the two-way memory effect. Although Wang and his associates assume that the structural transitions in this alloy are unique [14, 15] all essential features of present interest (in NiTi) correspond to those of the other alloys as shown, in particular, by the extensive work on NiTi by Wasilewski [16] and Nagasawa [17, 18].

In Table I we have compiled the alloy systems and phases which have so far been investigated with respect to one or more of the effects covered in the present paper. The dominant role played by bcc β-phases, as the parent phase, will be noted. This is due to distinct and comparatively large thermodynamic differences between the transformations of β-phases as compared, for instance, with the fcc → bcc (bct) transformations in ferrous alloys.

Several previous papers have dealt with the principles which govern the interactions of stresses with martensitic transformations. Much of this review is based on work by Nagasawa [18], Nakanashi *et al.* [19], Pops [20], Tas *et al.* [21], Warlimont and Delaey [22], Wasilewski [16], Wayman and Shimizu [23], Perkins [24], Bolling and Richman [4, 6] and Nakanishi [25]. Since a number of different terms are in use for the phenomena of present interest, Table II gives a survey of the terminology, corresponding terms being grouped together.

The following section deals with the structural basis and the microstructural changes associated with the transformations. In Part 2 the macroscopic mechanical behaviour and its characteristic variables are described. Part 3 deals with the thermodynamics and kinetics of the processes. Some of the open questions will also be discussed and considerations regarding practical applications of these effects summarized.

2. Structural and microstructural changes associated with the transformations

The formation of a martensite plate is accompanied by a macroscopic shape change. Although martensite formation is associated with this shape change which is characterized by an invariant plane strain minimizing the two-dimensional strain along the habit plane (plane AB in Fig. 5a), a three-dimensional strain is built up as the martensite plate grows into the third dimension. If the transformation occurs in a single crystal and if the habit plane extends from one free surface to the other free surface (Fig. 5a), i.e. a single-interface transformation, the two ends of the parent will be displaced with respect to each other. The amount and direction of the displacement will result from the thickness and relative orientation of the martensite plate. The formation of this single martensite plate can occur on cooling or on applying an external stress to the single crystal. The examples known in the literature are single-interface transformations upon cooling AuCd [26] and In–Tl single crystals [27] and Fe whiskers [28]. Examples of a single interface transformation by deformation are known for Cu–Al–Ni [29] and Ag–Cd [30] (Fig. 6). Upon heating the crystal or upon releasing the stress, the martensite plate may disappear by the same martensitic mechanism. The original shape of the crystal will be restored. Thus the single interface transformation which occurs during alternating cooling and heating or loading and unloading demonstrates the simplest mechanism for obtaining the two-way memory effect and pseudoelasticity.

If the starting material consists of a single

TABLE I

Alloys	Remarks	References
Thermoelastic		
Iron-platinum	Fe_3Pt (ordered)	37, 38
Copper-aluminium-nickel	$\beta_1 \rightarrow \gamma_1'$ bcc → orthor . (2H)	39, 68
Copper-zinc	$\beta_2 \rightarrow \beta_2'$ bcc → orthor. (3R)	40, 7, 41
Copper-zinc with ternary additions of Ni, Ag, Au, Cd, In, Ga, Si, Ge, Sn, Sb	$\beta_2 \rightarrow \beta_2'$ bcc → orthor. (3R)	42, 43
Ag–Cd	$\beta_2 \rightarrow \gamma_2'$ bcc → orthor. (2H)	44, 45, 30
Ti–Ni		46
Au–Cu–Zn		47, 19
Ni–Al		71
Pseudoelastic		
Cu–Zn	β_2 (bcc) → β_2' (orthor. 3R)	48, 49
Cu–Zn–X	β_2 (bcc) → β_2' (orthor. 3R)	20
Cu–Zn–Sn	β_2 (bcc) → orthor.	50, 20
Cu–Al–Ni	β_1 (bcc) → γ_1' (orthor. 2H)	35, 51, 52, 53, 69
Cu–Al–Mn	β_1 (bcc) → γ_1' (orthor. 2H)	69
Ag–Cd	β_2 (bcc) → γ_2' (orthor. 2H)	30, 73
Au–Cd	β_2 (bcc) → γ_2' (orthor. 2H)	11, 74
CuAuZn	β_3 (bcc) → β_3' (orthor. 3R)	47, 19
Fe_3Be	fcc → tetragonal (ordered bcc)	4
Fe_3Pt	fcc → tetragonal	
In–Tl	fcc → tetragonal	27, 2
Ni–Ti	bcc → orthor.	54
Ti–Ni		16
Au–Cu–Zn	β_2 (bcc) → β_2'' (2H + 18R)	19
Cu–Sn	β_1 (bcc) → γ_1' (orthor. 2H)	75
Shape memory effect		
In–Cd		55
Ti–Nb	β → orthor.	56
304 stainless steel	fcc → bcc α' fcc → hcp ϵ'	57
Ni–Al	bcc → CuAuI type	58
Cu–Zn		59, 60, 67
Ag–Cd	bcc → orthor.	30, 72
Fe–Ni		61
Ni–Ti		17
Cu–Al		62, 33, 21
Cu–Al–Ni		35, 60, 63
Ti–Ni		16, 64
Au–Cd		11, 74
In–Tl		65
Fe–Pt		38
Au–Cu–Zn		19
Cu–Zn–Si		66
Cu–Zn–Sn		50
Cu–Sn		75
Fe–Mn–C		76
Two-way shape memory effect		
In–Tl	fcc → tetragonal	27
Ti–Ni		13, 16
Cu–Al		21
Fe–Mn–C		77

TABLE II A comparative compilation of terms

Terms used in this paper	Terms used in other papers	References
Thermoelastic* (transformation)	thermoelastic (transformation)	39
Pseudoelasticity	superelasticity	53
	metalelasticity	19
	rubberlike behaviour†	2
	stress-induced pseudoelasticity (STRIPE)	20
	elastic shape memory effect	21
	ferroelasticity	11
	anelastic strain recovery	16
	anomalous elongation due to martensitic transformation	52
Shape memory effect	plastic shape memory effect	21
	Marmem	23
	strain memory effect	35
	annilation effect of martensitic shears	61
Two-way shape memory effect	reversible linear change on transformation	21, 16

*It should be noted that this term is not identical to the term *thermoelastic effects* used in the general thermodynamic treatment of crystal properties (e.g. [68]).

†Mainly used in referring to pseudoelasticity by reorientation.

martensite plate (Fig. 5b) the shape change can be achieved by applying a shear stress in such a way that the internal interfaces, which may either be twin boundaries or interfaces separating two different stacking sequences, move. One of the two twin variants or stacking variants will shrink while the other is growing resulting in the shown macroscopic shape change. Wayman and Shimizu [23] have stated the necessary conditions for the shape memory effect as follows:

(1) that the martensite is thermoelastic in nature,

(2) that the lattice invariant deformation occurs by twinning rather than by slip,

(3) that the martensite is formed from an ordered parent phase.

They also mention that "the bulk of the experimental evidence indicates that all marmem alloys consist of internally twinned martensites". However, Table I contains ample evidence of alloys whose internal structures do not contain twins but the structure of the martensite corresponds to an array of close-packed planes with a well defined, complicated stacking sequence (for example 3R). This complicated structure can, in

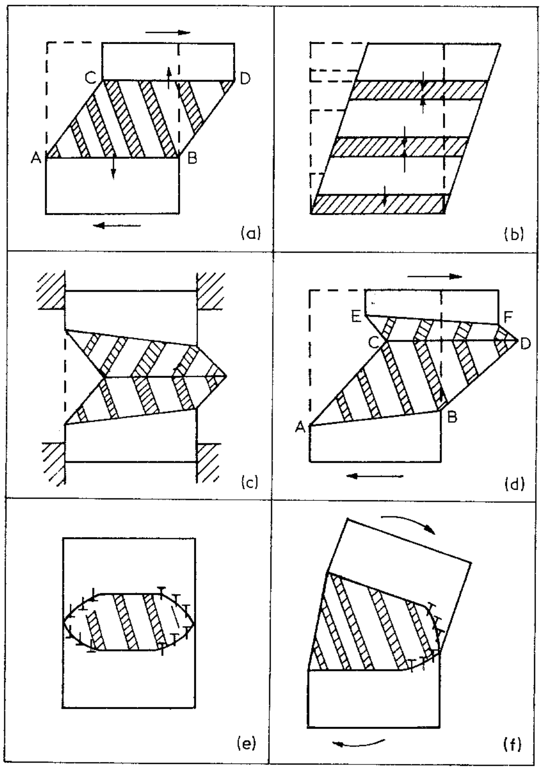

Figure 5 Changes introduced upon application of external stress. (a) Movement of the interfaces AB and CD giving rise to the increase of the volume of martensite. (b) movement of the boundaries in between the different twin related variants. (c) and (d) changes in the nature of variants formed in a self-accommodating manner. The boundary CD is displaced on application of external shear stress, (e) and (f) dislocations associated with the tapered end of the martensite and the bending caused.

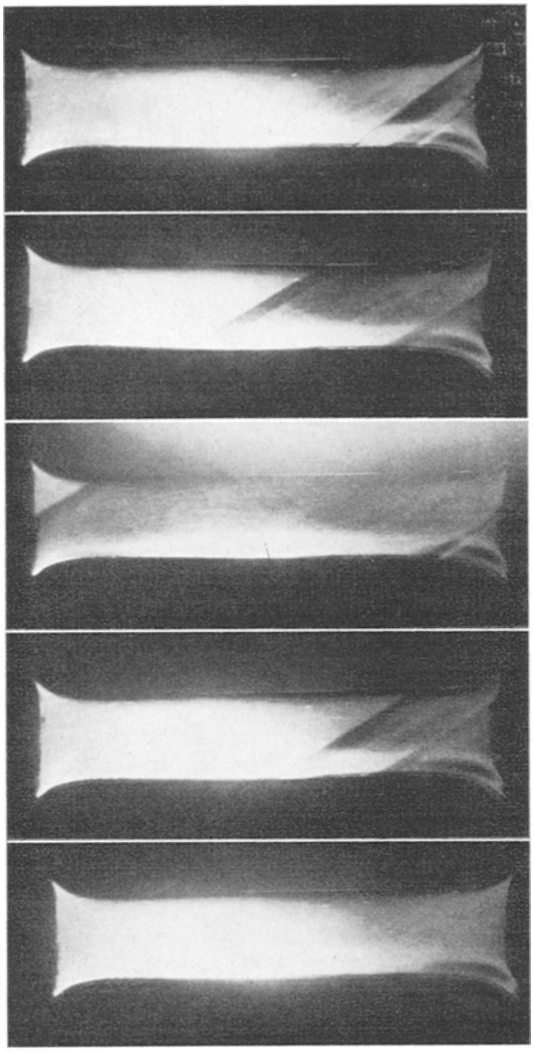

Figure 6 Single interface transformation in a single crystal of Ag–45 at.% Cd on application of an external stress. The figures show the increase in the amount of martensite with increase in stress (a → c) and later the decrease in the martensite upon unloading. The figures correspond to the tensile curve represented in Fig. 3 [30].

fact, be thought to be derived from fcc stacking with introducing stacking faults regularly, but this does not mean that irreversible slip is associated with the martensite formation. Although the atomic movements taking place during the bcc → 3R transformation are not known, the 3R-structure can also be derived from the bcc by simple shear and shuffle [70]. If subjected to an applied stress the pre-existing 3R-martensite plates grow and shrink (see Fig. 10), and in a completely martensitic (3R) sample, which shows perfect memory, the martensite plate boundaries move (see Fig. 9). These observations together with those mentioned in the literature prove that several martensites whose lattice-invariant deformation is not associated with twinning show perfect pseudoelasticity and shape memory effect. Hence twinning as lattice invariant deformation is not a necessary condition.

These examples show clearly that martensite

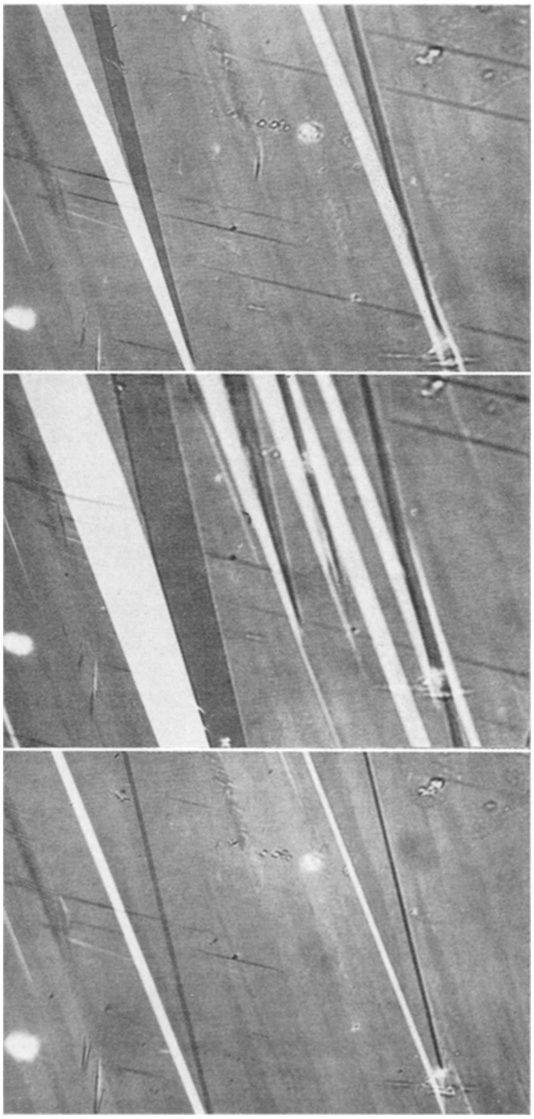

Figure 7 The formation of a stress-accommodating martensite plate group in a pseudoelastic material.

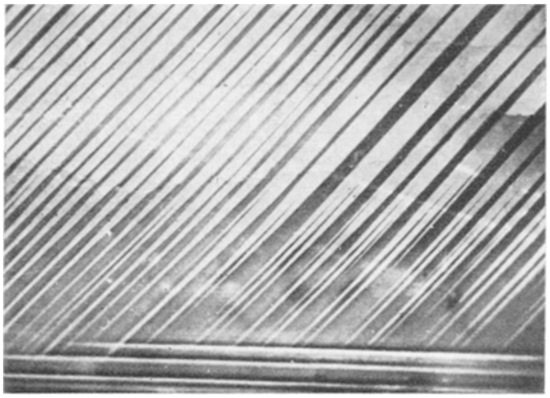

Figure 8 The tapering of stress-induced martensite plates. White, martensite; grey or black, the untransformed parent phase (β–Cu–Zn–Al).

formation and reorientation are deformation mechanisms similar to mechanical twinning. Under normal conditions, i.e. no single interface transformation and zero external stress, the formation and reverse transformation of martensite will not produce an external shape change of the specimen because differently oriented martensite plate variants form in such a way that the individual shape changes compensate each other. A particular mode of compensation of the shape changes is given by the self-accommodating character of the martensite formation in certain alloy systems. An idealized representation is given in Fig. 5c. The parent phase above and below the martensite plates is kept fixed, and in order to minimize the strains and stresses, two or more out of the twenty-four martensite plate variants will grow together as a self-accommodating martensite plate group [31]. Fig. 1 shows clearly how martensite plate variants grow together, forming a self-accommodating group.

If the sample is not kept fixed but strained by a shear stress as shown in Fig. 5d, the martensite plate group tends to minimize the internal stresses by changing the amount of the two variants present. This means that the boundary (CD) between the two martensite plates moves or that the habit plane (AB) moves faster than the habit plane (EF). The series of photographs given in Fig. 7 illustrates this behaviour clearly.

If the martensite is in contact with the matrix phase, the nature of the internal structure of the martensite plates will remain unchanged by the external stresses and strains, because the

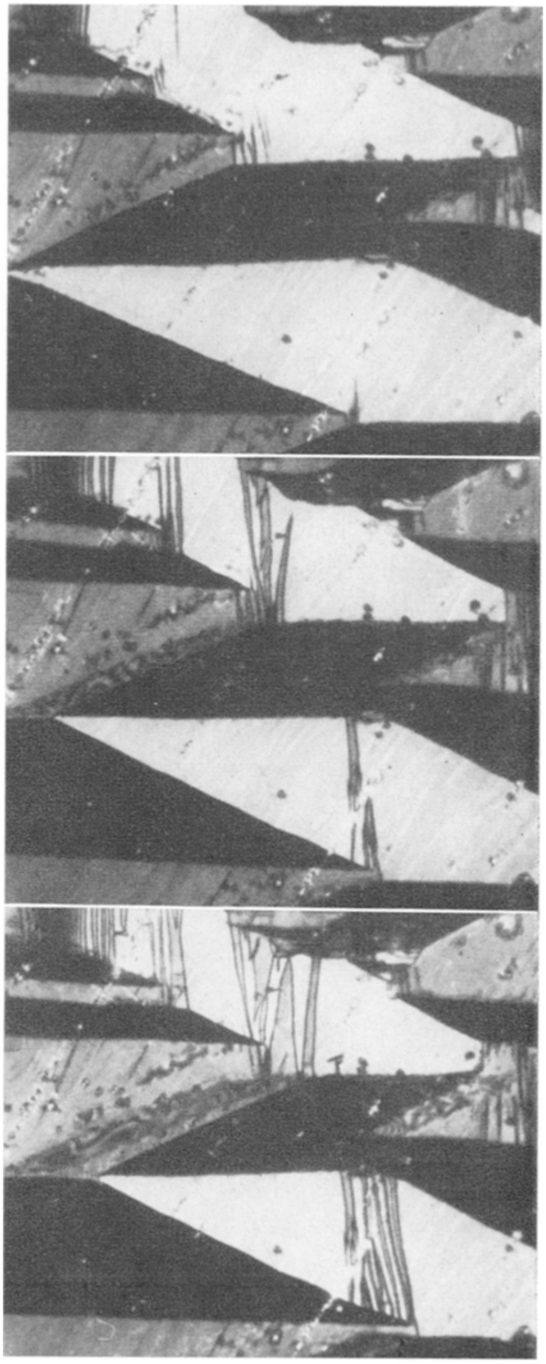

Figure 9 Reorientation of martensite by movement of already existing martensite plate boundaries (A and B) while applying an external stress (Cu–Zn–Al–Ni alloy).

invariant plane strain conditions must remain satisfied at the habit planes. But the external stresses necessary to move the interfaces, may

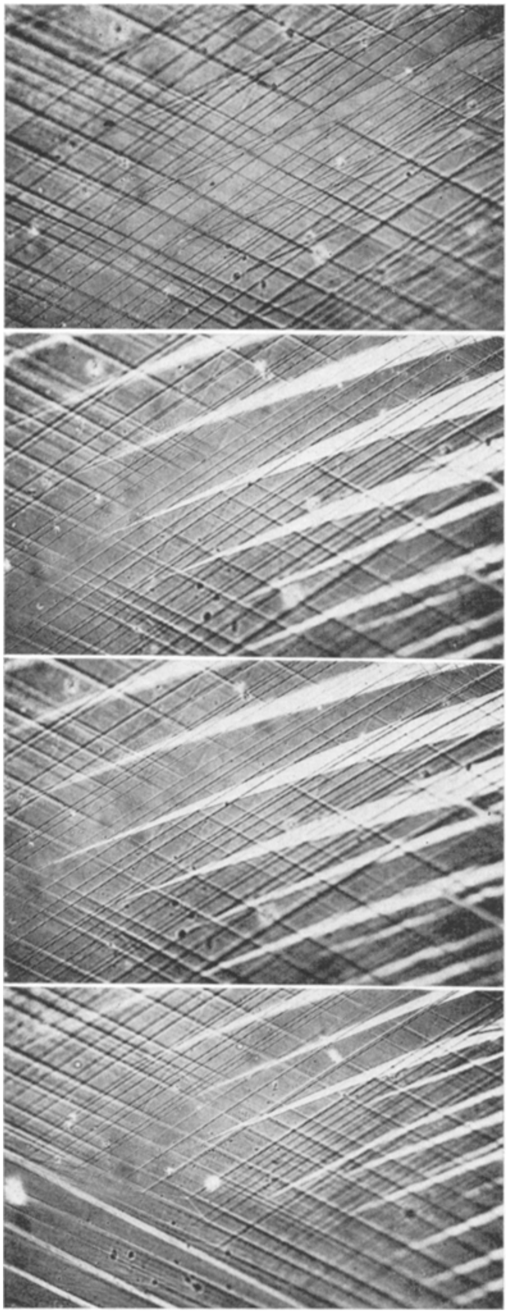

Figure 10 A series of micrographs showing pseudoelastic reorientation (Cu–Zn–Al).

exceed the yield stress of the matrix phase. Slip will then occur in one or both of the phases, altering the boundary conditions at the habit plane. It has been shown, in the case of Fe—Ni alloys that, as soon as slip has occurred in the matrix, the relative proportions of twins in each martensite plate are changed, thereby causing a rotation in the habit plane [32].

The martensite plate does not always belong to a self-accommodating group of plates, but, as Fig. 5e represents, the martensite plate may be tapered at both ends. Because at these tapered ends the invariant plane strain condition is no longer fulfilled, dislocations will be present at the end of the tapers. If the martensite plate shows a taper only at one end as shown in Fig. 5f, the specimen will be bent or, conversely, by applying bending stresses, the martensite can be made to taper at one end. The tapering of martensite plates in stressed material is shown in Fig. 8.

If the specimen is fully martensitic and consists of self-accommodating groups of martensite plates, then on stressing, movements of existing martensite plate boundaries (Fig. 9) or creation of new ones (Fig. 10) together with changes in the internal structure will occur. Because no untransformed parent phase is present, no invariant plane strain conditions need to be satisfied, only a three-dimensional strain minimization is necessary. The maximum shape change that can then be achieved is limited by the Bain strain [33].

Starting from a single crystal of β (i.e. copper-aluminium), martensite forms during quenching in a completely self-accommodating way, resulting in a zero external shape change. The martensite has a close-packed structure with a stacking sequence close to ABCBCACAB. Upon stressing, the amount of the different variants present in the original β single crystal is changed drastically and only a few variants are left out. On further stressing the martensite, the structure changes to either fcc or hcp depending on the orientation of the tensile axis. The single crystal of β has thus been transformed to a strongly textured and nearly unique orientation of the fcc or hcp stress-induced phase. The so-found fcc or hcp structure can be regarded as a structure resulting from the lattice (Bain) strain only. The maximum possible strain that can be obtained was calculated with respect to the crystal orientation and the experimental values agreed well with the theoretical predictions (Fig. 11a and b) [31, 33].

In the case of Ag–45 at. % Cd alloy single crystals, stress-induced martensite was found to form directly from the β-phase at temperatures well above the M_s temperature. This stress-induced martensite formed gradually at an

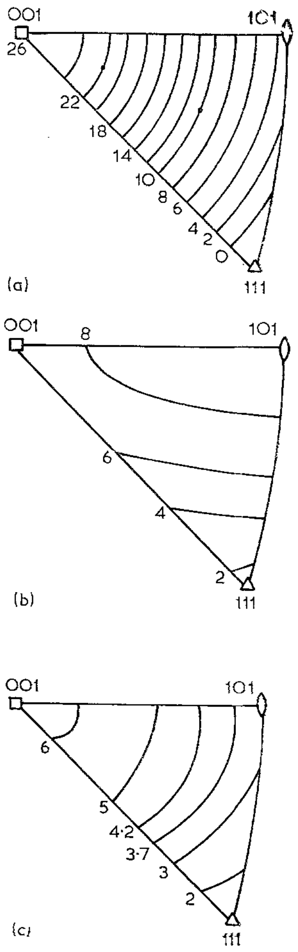

Figure 11 (a) and (b). Theoretical elongations (%) possible on going from a 3R structure to face-centred cubic or hexagonal close-packed structure in the case of Cu–Al alloy single crystals, for different orientations of tensile axis. Calculations are equivalent to going from β → fcc or hcp since the martensite forms in a completely self-accommodating way. (c) Theoretical calculations in the case of Ag–45 at.% Cd alloy single crystals, for different orientations of the tensile axis, calculated on the basis of the total shape deformation associated with the transformation obtained from the phenomenological theory.

almost constant level of stress and the amount of linear strain obtained agreed well with the value calculated using the phenomenological theory of martensitic transformations [30]. Fig. 11c gives the calculated values of the linear strain as a function of the orientation of the tensile axis of the specimen. In this case the values were obtained without taking into account the rigid body rotation (an infinitesimally small disc of the tensile specimen was considered to transform); this is in contrast to the Cu–Al case where transformation of the complete specimen was considered. Calculations of the amount of elongation have also been performed for the case of pure reorientation of the internally twinned hcp Cu–Al–Ni martensite [34, 35].

The reverse transformation (i.e. from martensite to the parent phase) proceeds in most cases along the same path as the initial transformation, because in these cases the symmetry of the low temperature (martensite) phase is lower than the symmetry of the parent phase. Therefore, whilst a large number of crystallographically equivalent martensite plates can form in the parent phase (a maximum of twenty-four for a cubic parent phase) only a limited number of different orientations of the parent phase can form from the martensite crystal. In some cases only one orientation of the parent phase can be formed from the martensite crystal, an orientation indentical to the orientation of the original parent phase. This is especially so in the case where the parent phase possesses long range order, since this imposes additional restrictions as well [23]. Even when the formation of several parent orientations is possible, transformation to the original parent orientation will still be preferred because in this way no new accommodation stresses are introduced and no new interfaces need be created. This effect will be very strong for a self-accommodating group of martensite plates in which a strong mechanical coupling exists among the different martensite variants.

As mentioned earlier, the effect of an external tensile stress can either be to form stress-induced martensite (Fig. 12a) or to cause a reorientation of existing martensite variants. However, the restrictive action of the end grips restrains free movement of the sample and causes problems of lattice bending and rotation adjacent to the grips as in the plastic deformation process. The variant of martensite formed on the application of external stress depends on the nature of the external stress. Similarly, when more than one variant is present, the reorientation will not be identical in tension and in compression. This is represented in Fig. 12b, in which the variants 1 and 2 are preferred under tensile and compressive stresses respectively. These arguments are based on the glissile nature of the interface between the two variants. A typical example of this behaviour is given in Fig. 9.

In simple bending, however, the top and the bottom layers are under tension and compression, respectively, and so this can be considered as a combination of the two cases discussed above. Fig. 12c is a schematic representation of the processes involved. The specimen can be considered as being made up of a number of thin layers, which should be extended or compressed depending on their position with respect to the neutral plane (i.e. layers above this plane are extended and those below compressed). If each strip is now elastically bent into cylindrical shape, the strips can be fitted together again without any long-range stress, provided that the change in length of any given layer is proportional to the distance of that layer from the neutral plane. The net effect is that the interfaces between the variants are bent as shown

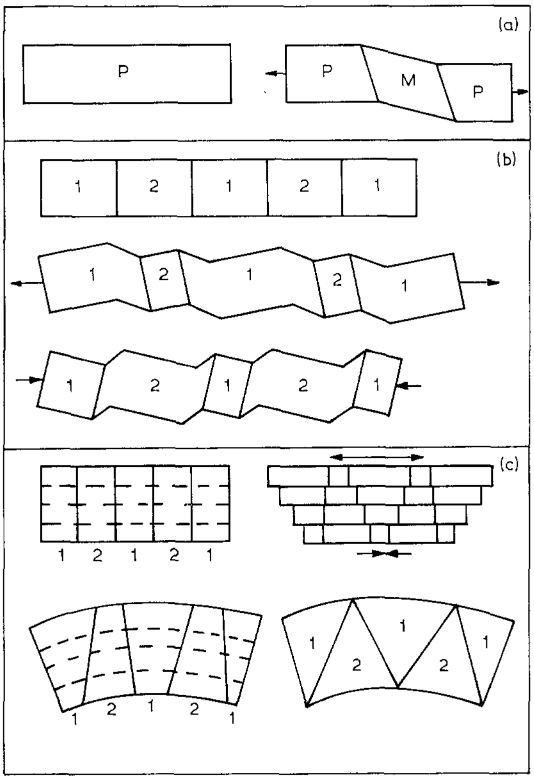

Figure 12 (a) Formation of stress-induced martensite upon application of an external tensile stress. (b) Changes involved upon tension and compression when the specimen is composed of two different variants. Note that variant 1 and 2 are favoured in tension and compression respectively. (c) Schematic representation showing the changes involved on bending a specimen containing two variants.

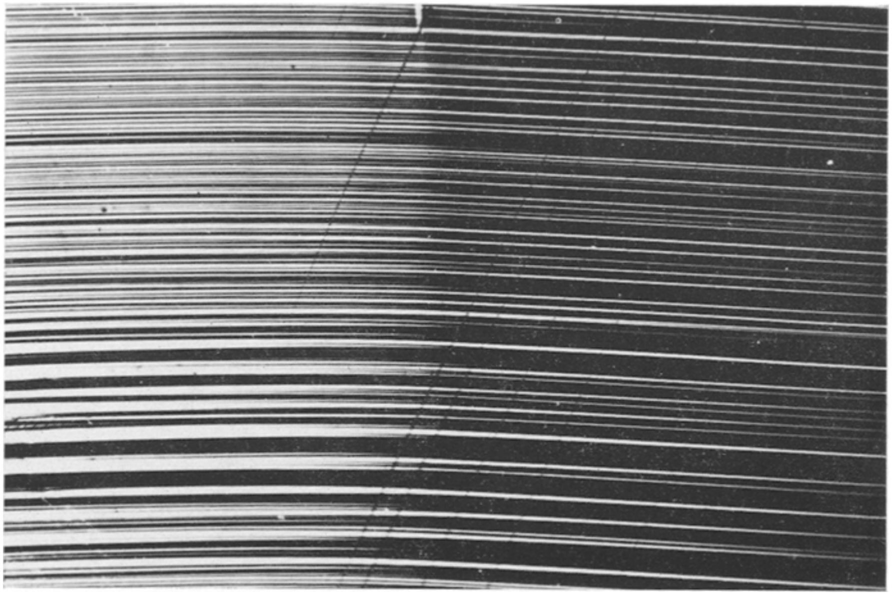

Figure 13 Bending associated with the tapered martensite plates. The black areas correspond to the untransformed β–Cu–Zn–Al phase whilst the white lamellae are the martensite plates.

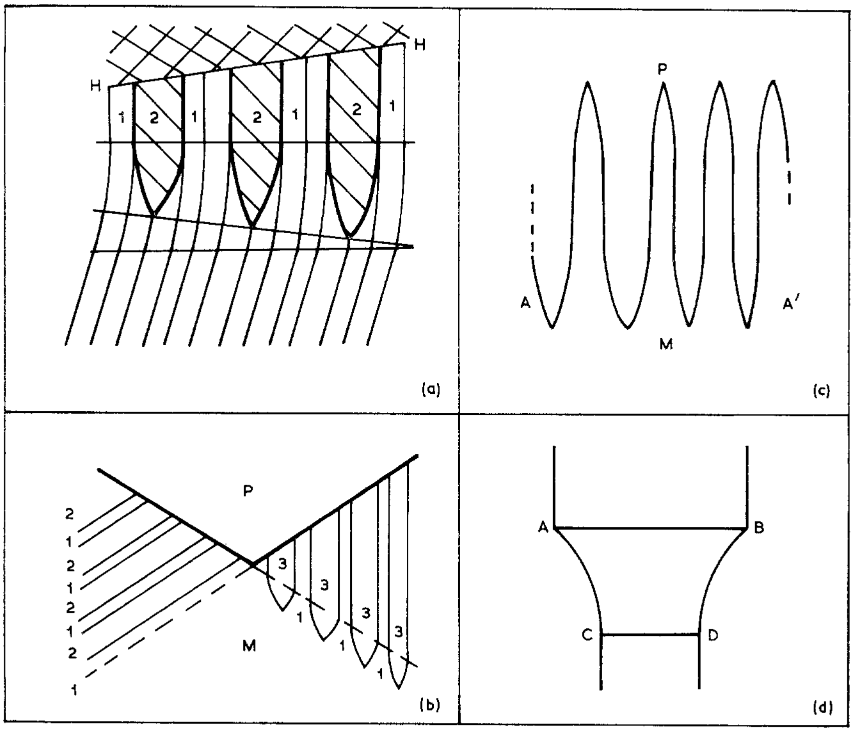

Figure 14 Schematic representation of taper formation on deformation of (a) martensite composed of two variants, into a single martensite orientation [27]. Parent and martensite phase have a common interface HH (habit plane); (b) a single parent orientation into a single martensite orientation by combined movement of two interfaces [27]; (c) a single parent orientation into a single martensite orientation by the combined movement of tapers and of reorientation. The parent and matrix plane have several planes in common (see Fig. 8); (d) a tensile sample. Parts AC and BD represent the bending.

in the figure. The relative amounts of the two variants present vary as we go down from the top to the bottom layer. When the twin planes are parallel to the axis of the specimen, no bending is possible. The bending of the interface plane can be considered to be associated with twinning dislocations in this interface. The bending which is associated with the position of the tapered martensite plate ends is illustrated in Fig. 13.

Basinski and Christian [27] considered the double-interface transformations leading to the formation of a single crystal of martensite from a single crystal of the original beta phase. Similar arguments can be invoked to explain the transition zone observed by Hornbogen *et al.* [36]. In this transition region one can consider a diffuse interface arising from tapering of the twin variants present. The formation of a single crystal of martensite from a single crystal of the matrix phase is illustrated in Fig. 14 following the approach of Basinski and Christian [27].

A single crystal of the parent beta phase can transform to a single crystal of martensite according to Fig. 14a to c. Fig. 14a shows the transition region between the martensite and the parent phase. This consists of a number of twins which taper at distances away from the interface. This tapering is associated with bending of the lattice, while at the habit plane (HH) the correct twin ratio, as dictated by the theory is maintained. In Fig. 14b, the transformation is achieved by the movement of two interfaces resulting in a single orientation of the martensite [27] whilst another situation is envisaged in Fig. 14c. The transition from martensite to the parent is gradual with martensite tapering out as small plates. This results in bending and elongation. Farther away from this interface, the internal twins at AA′ can disappear as represented in Fig. 14a (refer to Fig. 5b) and this also gives rise to a single orientation of martensite. When elongation is achieved in the case where the specimen is gripped at the ends, there is always the problem of bending associated with the transition region. This is easily accommodated by the tapering associated in the martensite plates (Fig. 14c).

The importance of bending is clearly demonstrated in the experiments on polycrystalline specimens. It is often seen that several parallel martensite plates grow instead of a single martensite plate of the same volume forming. This is probably due to the fact that several martensite plates are able to accommodate the bending moments associated with the transformation stresses. The taper associated with each plate can account for the bending and several parallel individual plates can easily accommodate more bending compared to a single martensite plate of comparable volume.

References

1. J. BRETTSCHNEIDER and H. WARLIMONT, *Z. Metallk.* **59** (1968) 740.
2. M. W. BURKART and T. A. READ, *Trans. Met. Soc. AIME* **197** (1953) 1516.
3. J. R. PATEL and M. COHEN, *Acta Met.* **1** (1953) 531.
4. G. F. BOLLING and R. H. RICHMAN, *ibid* **13** (1965) 709, 723, 745.
5. F. LAVES, *ibid* **9** (1965) 58.
6. R. H. RICHMAN and G. F. BOLLING, *Met. Trans.* **2** (1971) 2451.
7. A. B. GRENINGER and V. G. MOORADIAN, *Trans. Met. Soc. AIME* **128** (1938) 337.
8. G. V. KURDYUMOV and L. G. KHANDROS, *Dokl. Akad. Nauk. SSSR* **66** (1949) 211.
9. E. SCHEIL, *Z. Anorg. Allg. Chem.* **207** (1932) 21.
10. E. HORNBOGEN and G. WASSERMANN, *Z. Metallk.* **47** (1956) 427.
11. L. C. CHANG and T. A. READ, *Trans. Met. Soc. AIME* **191** (1951) 47.
12. H. K. BIRNBAUM and T. A. READ, *ibid* **218** (1960) 662.
13. W. J. BUEHLER, J. V. GILFRICH and R. C. WILEY, *J. Appl. Phys.* **34** (1963) 1475.
14. F. E. WANG, *ibid* **43** (1972) 92.
15. F. E. WANG, S. J. PICKART and H. A. ALPERIN, *J. Appl. Phys.* **43** (1972) 97.
16. R. J. WASILEWSKI, *Met. Trans.* **2** (1971) 2973.
17. A. NAGASAWA, *J. Phys. Soc. Japan* **31** (1971) 136.
18. *Idem, Phys. Stat. Sol.* (*a*) **8** (1971) 531.
19. N. NAKANISHI, Y. MURAKAMI and S. KACHI, *Scripta Met.* **5** (1971) 433.
20. H. POPS, *Met. Trans.* **1** (1970) 251.
21. H. TAS, L. DELAEY and A. DERUYTTERE, *J. Less-Common Metals* **28** (1972) 141.
22. H. WARLIMONT and L. DELAEY, *Prog. Mat. Sci.* **19** (1974).
23. C. M. WAYMAN and K. SHIMIZU, *Met. Sci. J.* **6** (1972) 175.
24. J. PERKINS, to be published. *Met. Trans.* **4** (1973) 2709.
25. N. NAKANISHI, *Mémoirs of the Kônan University, Science Series* **15** Art. 77 (1972).
26. D. S. LIEBERMAN, "Phase Transformations", (A.S.M., Cleveland, Ohio, 1970) p. 7.
27. Z. S. BASINSKI and J. W. CHRISTIAN, *Acta Met* **2** (1954) 148.
28. R. P. ZERWEKH and C. M. WAYMAN, *ibid* **13** (1965) 99.
29. K. OTSUKA, M. TAKAHASHI and K. SHIMIZU, *Met. Trans.* **4** (1973) 2003.
30. R. V. KRISHNAN and L. C. BROWN, *Met. Trans.* **4** (1973) 432; and R. V. KRISHNAN, Ph.D. Thesis, University of British Columbia, Canada (1971).
31. H. TAS, L. DELAEY and A. DERUYTTÈRE, *Met. Trans.* **4** (1973) 2833.
32. H.-P. SATTLER and G. WASSERMANN, *J. Less-Common Metals* **28** (1972) 119.
33. H. TAS, L. DELAEY and A. DERUYTTÈRE, *Scripta Met.* **5** (1971) 1117; *Z. Metallk.* **64** (1973) 855, 862.
34. Z. S. BASINSKI and J. W. CHRISTIAN, *Acta Met.* **2** (1954) 101.
35. K. OISHI and L. C. BROWN, *Met. Trans.* **2** (1971) 1971.
36. E. HORNBOGEN, A. SEGMULLER and G. WASSERMANN, *Z. Metallk.* **48** (1957) 379.
37. D. P. DUNNE and C. M. WAYMAN, *Met. Trans.* **4** (1973) 137.
38. C. M. WAYMAN, *Scripta Met.* **5** (1971) 489.
39. G. V. KURDJUMOV, *J. Techn. Phys. U.S.S.R.* **18** (1948) 999.
40. H. POPS and T. B. MASSALSKI, *Trans. Met. Soc. AIME* **230** (1964) 1662.
41. D. HULL and R. D. GARWOOD, "The Mechanism of Phase Transformations in Metals" (The Institute of Metals Monograph and Report Series No. 18, London, 1956) p. 219.
42. H. POPS, *Trans. Met. Soc. AIME* **236** (1966) 1532.
43. *Idem, ibid,* **239** (1967) 756.
44. R. MÜLLER and H. WARLIMONT, to be published.
45. R. V. KRISHNAN and L. C. BROWN, *Met. Trans.* **4** (1973) 1017.
46. K. OTSUKA, T. SAWAMURA, K. SHIMIZU and C. M. WAYMAN, *ibid* **2** (1971) 2583.
47. Y. MURAKAMI, H. ASANO, N. NAKANISHI and S. KACHI, *Jap. J. Appl. Phys.* **6** (1967) 1265.
48. J. E. REYNOLDS and M. B. BEVER, *Trans. Met. Soc. AIME* **194** (1952) 1065.
49. E. F. SUONINEN, R. M. GENEVRAY and M. B. BEVER, *Idem,* **206** (1956) 283.
50. J. D. EISENWASSER and L. C. BROWN, *Met. Trans.* **3** (1972) 1359.
51. W. A. RACHINGER, *J. Australian Inst. Metals* **5** (1960) 114.
52. I. A. ARBUZOVA and L. G. KHANDROS, *Fiz. Metal. Metalloved.* **17** (1964) 390.
53. W. A. RACHINGER, *Brit. J. Appl. Phys.* **9** (1958) 250.

54. W. J. BEUHLER and W. B. CROSS, *Wire J.* **2** (1969) 41.
55. N. NAKANISHI, Y. MURAKAMI, T. MORI, S. KACHI and S. MIURA, to be published.
56. C. BAKER, *Mat. Sci. J.* **5** (1971) 92.
57. K. ENAMI, S. NENNO and Y. MINATO, *Scripta Met.* **5** (1971) 663.
58. K. ENAMI and S. NENNO, *Met. Trans.* **2** (1971) 1487.
59. E. HORNBOGEN and G. WASSERMANN, *Z. Metallk.* **47** (1956) 427.
60. K. OTSUKA, K. SHIMIZU, J. CORNELIS and C. M. WAYMAN, *Scripta Met.* **6** (1972) 377.
61. A. NAGASAWA, *J. Phys. Soc. Japan* **30** (1971) 1505.
62. A. NAGASAWA and K. KAWACHI, *ibid* **30** (1971) 296.
63. K. OTSUKA and K. SHIMIZU, *Scripta Met.* **4** (1970) 469.
64. C. M. WAYMAN, J. CORNELIS and K. SHIMIZU, *ibid* **6** (1972) 115.
65. A. NAGASAWA, *J. Phys. Soc. Japan* **30** (1971) 1200.
66. D. V. WIELD and E. GILLAM, *Scripta Met.* **6** (1972) 1157.
67. W. ARNEODO and M. AHLERS, submitted to *Scripta Met.*
68. I. A. ARBUZOVA, V. S. GAVRIL'YUK and L. G. KHANDROS, *Fiz. Metal. Metalloved.* **30** (1970) 181; **27** (1969) 1126.
69. A. L. KUPOREV and L. G. KHANDROS, *Fiz. Metal, Metalloved.* **32** (1971) 214.
70. L. DELAEY, J. VAN PAEMEL and T. STRUYVE, *Scripta Met.* **6** (1972) 507.
71. Y. K. AU and C. M. WAYMAN, *Scripta Met.* **6** (1972) 1209.
72. H. C. TONG and C. M. WAYMAN, *ibid* **7** (1973) 215.
73. S. MIURA, T. MORI and N. NAKA-NISHI, *Scripta Met.* **7** (1973) 697.
74. N. NAKANISHI, T. MORI, S. MIURA, Y. MURAKAMI and S. KACHI, *Phil. Mag.* **28** (1973) 277.
75. S. MIURA, S. MAEDA and N. NAKANISHI, to be published.
76. H. SCHUMANN, *Kristall und Technik*, **9** (1974) 281.
77. *Idem, ibid*, **9** (1974) 33.

Received 1 February and accepted 19 February 1974.